Math notes

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1 Introduction

Here I will fill in notes from lecture and additional topics for you to review. There is a section for notes from lecture, along with subsections for the dates, and a section of general topics and descriptions to review that will assist you in the course.

2 General topics to know

- Greek letters (see handouts page)
- Norms
- Triangle inequality
- Trigonometric identities
- Geometric rules
- Solving systems of equations
- Solving quadratic equations

3 Notes from lecture

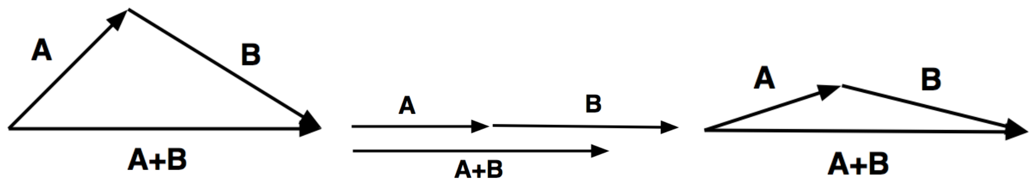
3.1 Tu Sept. 2, 2014

3.1.1 Triangle inequality

The triangle inequality is an important relationship. Given two vectors $\vec{A}$ and $\vec{B}$, the following relationship holds:

$$|\vec{A} + \vec{B}| \leq |\vec{A}| + |\vec{B}| \tag{1}$$

Geometrically and intuitively this makes sense, as we can demonstrate below in Figure (1):



(a) (b) (c)

Figure 1: (a) Shows a triangle formed by A, B, and A+B. (b) Note that if you take the vectors and put them parallel to each other (but don't change their length), B placed at the tip of A will always be at least as long as the vector formed by adding A+B. (c) Shows how, as the vectors get closer and closer to parallel vectors, we come to the case of —A—+—B—=—A+B— - that is, when the two vectors are parallel to each other.

3.1.2 Norms

Here we discussed norms briefly. Now norms have a lot more to them than we will go into in this course. I just want you to be aware that there are many types, and generally since we are dealing with Euclidian space in this course, we are dealing with 2-norms, as described below.

The book uses norms as a general term, and we discussed the fact that norms more generally are given as n-norms in n-dimensional spaces (also commonly referred to as 'p-norms') by (with a real number $n \geq 1$). The 2-norm is a special case, also referred to as the Euclidian norm, because it is in 2D Euclidian space. It is also commonly known as the magnitude in 2D space. Here is a more general example in n-dimensional space (real numbers):

$$||\vec{x}||_n = \left(\sum_i (x_i)^n \right)^{1/n} \tag{2}$$

For example, the 3-norm of a 2D vector x is given by

$$||\vec{x}||_3 = \sqrt[3]{(x_1)^3 + (x_2)^3} \tag{3}$$

We also mentioned that the absolute value operation is actually not the same as a norm (the book refers to 2-norms as simply norms, so here we imply a 2-norm), and is shown with a single 'pipe' on either side of the quantity:

$$|\vec{x}| = (|x_1|, |x_2|, ..., |x_n|) \tag{4}$$

The absolute value of a vector is a vector of the same dimensionality, with everything made to be positive (ie any negative vector components are reflected to the positive side. For example, if $\vec{x} = (-1, 1)$, then $|\vec{x}| = (1, 1)$). The norm of a vector is a scalar value. For example, given $\vec{x} = (2, 3)$,$||\vec{x}|| = (2^2 + 3^2)^{1/2} = \sqrt{13} \approx 3.61$.

Thought question : if you take the absolute value of a vector which points in the negative y-axis, what will it look like now?

To clear up a little confusion during the discussion, we discussed that an absolute value is basically a 1-norm for one-dimensional variables. A 1-norm is given by the sum of the absolute values of the elements of an n-dimensional vector.

$$||\vec{x}||_1 = \sum_i |x_i| \tag{5}$$

So the difference is the absolute value of a vector is still a vector, but a 1-norm is a scalar number. Generally up until now you have delt in math classes with single-dimensional variables. For this class we are dealing often with two and three-dimensions. In the future you might deal with thousands or more dimensional variables!

3.1.3 Fundamental properties of vector multiplication

Given the vectors A and B, and scalars s and t, the following hold true for vector multiplication,

$$0 \cdot A = A \tag{6}$$
$$1 \cdot A = A \tag{7}$$
$$(-1) \cdot A = A \tag{8}$$
$$(s + t) \cdot A = A \tag{9}$$
$$s(A + B) = sA + sB \tag{10}$$
$$s(tA) = (st)A \tag{11}$$