Useful Math Review notes

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1 Description

Here you will find some useful mathematical techniques with an example or two as relevant. This document will be updated as the semester progresses. Please check the date at the top against what you have on your hard drive.

2 Calculus

2.1 Limit laws

from Stewart, 3rd edition, page 61

Suppose c is a constant and the limits
$lim_{x->a} f(x)$ and $lim_{x->a} g(x)$
exist. Then

1. $lim_{x->a}[f(x) + g(x)] = [lim_{x->a} f(x)] + [lim_{x->a} g(x)]$
2. $lim_{x->a}[f(x) - g(x)] = [lim_{x->a} f(x)] - [lim_{x->a} g(x)]$
3. $lim_{x->a}[cf(x)] = c[lim_{x->a} f(x)]$
4. $lim_{x->a}[f(x) * g(x)] = [lim_{x->a} f(x)] * [lim_{x->a} g(x)]$
5. $lim_{x->a}[\frac{(x)}{g(x)}] = \frac{[lim_{x->a} f(x)]}{[lim_{x->a} g(x)]}$ if $[lim_{x->a} g(x)] \neq 0$
6. $lim_{x->a}[f(x)]^n = [[lim_{x->a} f(x)]^n$ where n is a positive integer
7. $lim_{x->a} c = c$

8. $lim_{x->a} x = a$

9. $lim_{x->a} x^n = a^n$ where n is a positive integer

10. $lim_{x->a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer (If n is even, we assume that a¿0.)

11. $lim_{x->a} \sqrt[n]{f(x)} = \sqrt[n]{lim_{x->a} f(x)}$ where n is a positive integer (If n is even, we assume that $lim_{x->a} f(x) > 0$

2.2 Definitions

from Stewart, 3rd edition, page 82

1. A function f is continuous at a number a if

   $lim_{x->a} f(x) = f(a)$

2. a function f is continuous from the right at a number a if

   $lim_{x->a^+} f(x) = f(a)$

   and f is continuous from the left at a if

   $lim_{x->a^-} f(x) = f(a)$

3. A function f is continuous on an interval if it is continuous at every number in the interval. (At an endpoint of the interval we understand continuous to mean continuous from the right or continuous from the left.)

2.3 Example

Show that the function f(x)=1-x is continuous on the interval [-1,1].

Solution: If $-1 < a < 1$, then using the limit laws, we have
$lim_{x->a} f(x) = lim_{x->a}(1 - x)$
$= 1 - lim_{x->a}(x)$
$= 1 - a = f(a)$

Thus by definition 1, f is continuous at a if $-1 < a < 1$. We must also calculate the right-hand limit at -1 and the left hand limit at 1.
$lim_{x->-1^+} f(x) = lim_{x->-1^+}(1 - x)$
$= 1 - lim_{x->-1^+}(x)$
$= 1 - (-1)$
$= 2 = f(-1)$

So f is continuous from the right at -1. Similarly,
$lim_{x->1^-} f(x) = lim_{x->1^-}(1 - x)$
$= 1 - lim_{x->1^-}(x)$
$= 1 - (1)$
$= 0 = f(1)$

So f is continuous from the left at 1. Therefore, according to Definition 3, f is continuous on [-1,1] (recall that the "[.]" brackets mean including those points).

2.4 Derivatives

$\frac{d}{dx}(c) = 0$ (where c is some constant)

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(sin(x)) = cos(x)$

$\frac{d}{dx}(cos(x)) = -sin(x)$

$\frac{d}{dx}(tan(x)) = sec^2(x)$

$\frac{d}{dx}(csc(x)) = -csc(x) * cot(x)$

$\frac{d}{dx}(sec(x)) = sec(x) * tan(x)$

$\frac{d}{dx}(cot(x)) = -csc^2(x)$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(e^u) = e^u * \frac{du}{dx}$ (where u is an arbitrary function)

$\frac{d}{dx}(ln(x)) = \frac{1}{x}$

$(cf)' = c(f)'$ (where c is a constant)

$(f - g)' = f' - g'$

$(f + g)' = f' + g'$

$(fg)' = f'g + g'f$

$(\frac{f}{g})' = \frac{f'g - g'f}{g^2}$

$\frac{d}{dx}(a^x) = a^x * ln(a)$ (where a is some constant)

3 Matrix math

Coming soon!

4 Sums of Series

4.1 Taylor Series

The Taylor Series allows us to represent a function with a polynomial function (referred to as a power series, in this particular application) which is an infinite dimensional sum. Typically this is just done to the first or second order in practice, and is assumed to be valid for a small area about the operating point (the point we choose to make the expansion about). When approximated, we say that we 'approximate' the function to the order specified.

The equation for a taylor series expansion is simply

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... \tag{1}$$

or compactly

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \tag{2}$$

so, for example, the taylor series expansion of log(x) to the second order about x_0 is

$$log(x) \approx log(x_0) + \frac{1}{x_0}(x-x_0) - \frac{1}{x_0^2}(x-x_0)^2 \tag{3}$$

Practice a few problems to get it in your mind again, ready for action!
Try approximating $f(x) = cos(x)$ to the third order about the point a.
Try approximating $f(x) = e^x$ to the fourth order about the point 5.

4.2 Fourier Series

You should look over fourier series. This is helpful.