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Beam Deflections by Discontinuity Functions

A Short Table of Discontinuity Functions

Name	Symbol	Definition
doublet	$\langle x-a \rangle^{-2}$	antiderivative of doublet function is delta function
[Dirac] delta	$\langle x-a \rangle^{-1}$	antiderivative of delta function is step function
step	$\langle x-a \rangle^0$	1 if $x > a$, else 0
ramp	$\langle x-a \rangle^1$	$x-a$ if $x > a$, else 0
parabolic ramp	$\langle x-a \rangle^2$	$(x-a)^2$ if $x > a$, else 0
.....		
n^{th} order ramp	$\langle x-a \rangle^n$	$(x-a)^n$ if $x > a$, else 0 ($n \geq 0$)

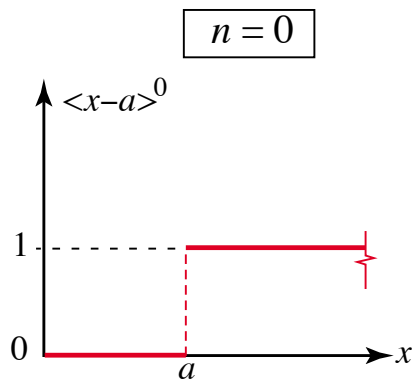
Useful integration formula for $n = 0, 1, 2, \dots$

$$\int_{x_0}^x \langle x-a \rangle^n dx = \frac{\langle x-a \rangle^{n+1}}{n+1} \quad \text{valid if } n \geq 0 \text{ and } x_0 \leq a$$

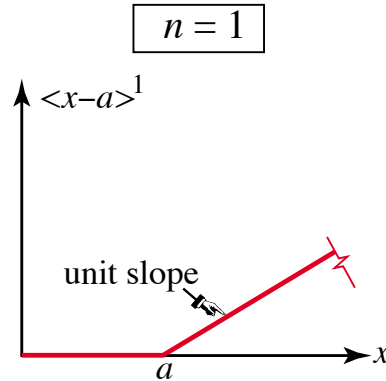
For beam problems where origin of x is at left end, x_0 is normally 0

If $n = -1$, integral is step function. If $n = -2$, integral is delta function.

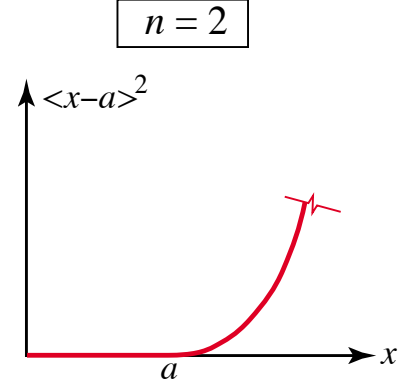
If n is nonnegative, we get ordinary functions that can be plotted as shown below



Step function, also called unit step and Heaviside function

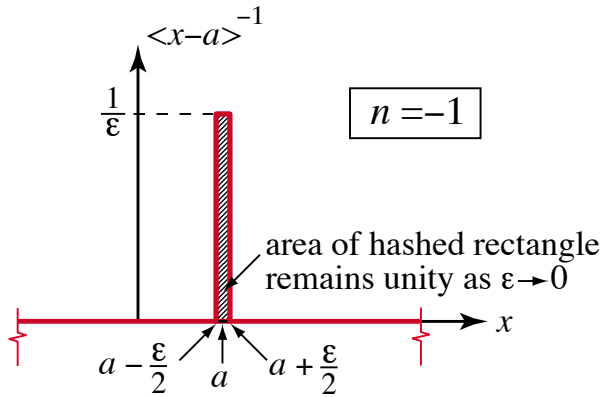


Unit ramp function

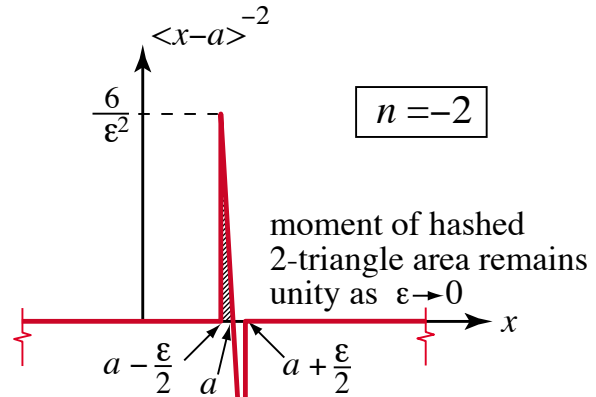


Parabolic unit ramp function

If n is negative, we get singularity functions that cannot be plotted - only visualized as limit of a function sequence

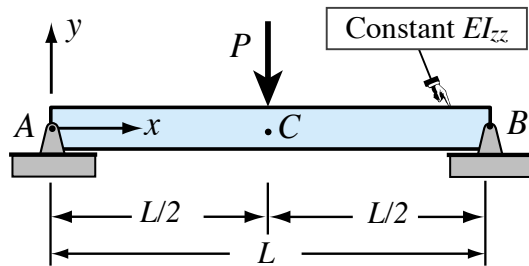


Delta function, also called Dirac delta (in Physics) and unit impulse function (in dynamics)

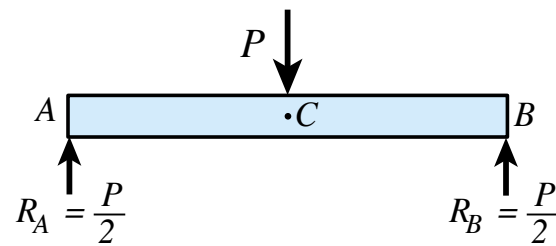


doublet function, also called dipole (in Physics) and double impulse (in dynamics)

Example 1: SS Beam Under Central Point Load

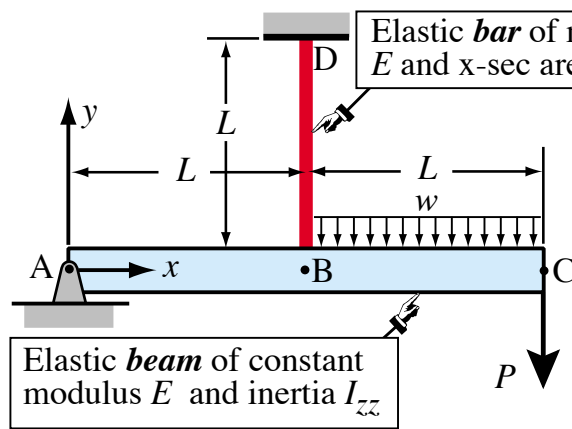


(a) Problem definition

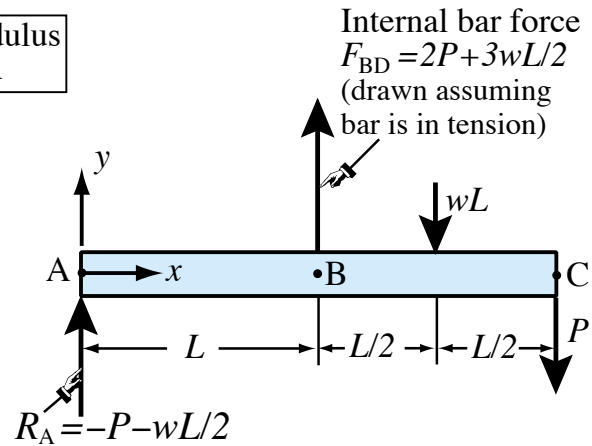


(b) FBD to get support reactions

Example 2: Hinged Beam Propped by Elastic Bar (A Variation of Problem 2 of Recitation 5)

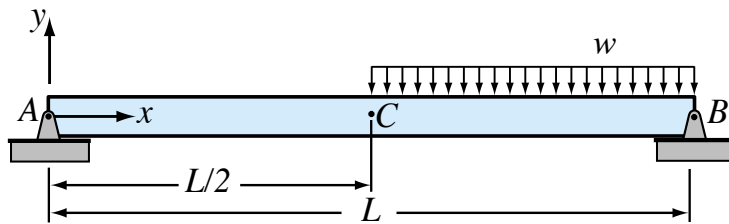


(a) Problem definition



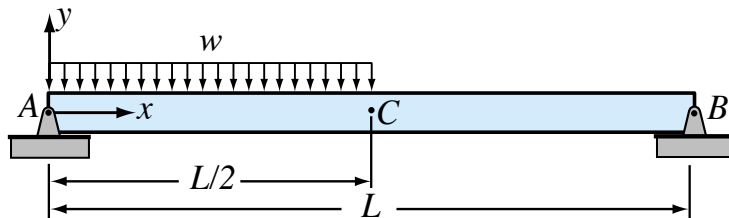
(b) FBD to get reaction and bar force

Expression in Terms of D.F. May Require Some Load Superposition Tricks



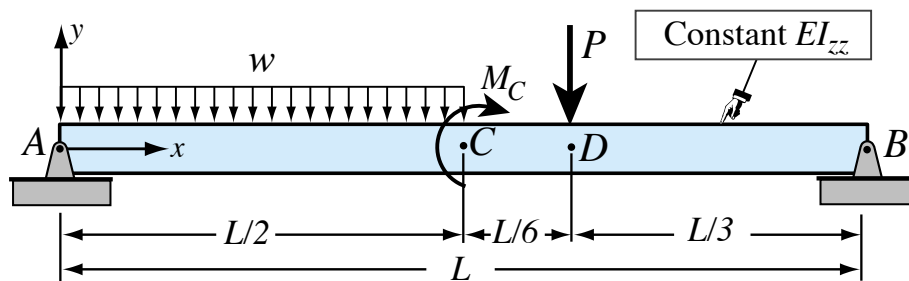
No problem here: step function starts at $x=L/2$, whence

$$p(x) = -w \langle x - L/2 \rangle^0$$

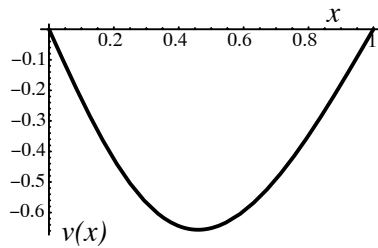


Houston, we got a problem. This is **not** a step function

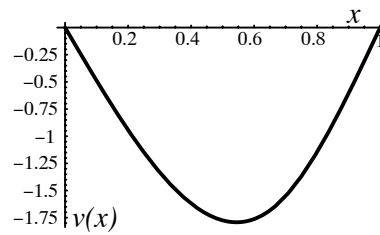
Example 3: SS Beam With Complicated Loading



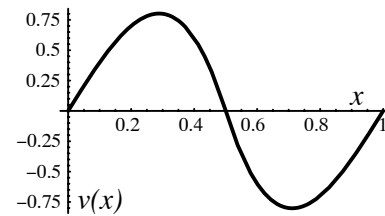
Example 3: Numerical Results For Deflection Function (obtained and plotted with *Mathematica*)



(a) Deflection curve for $w=100$,
 $P=0, M_C=0, EI_{zz} = 1$ and $L=1$



(b) Deflection curve for $w=0$,
 $P=100, M_C=0, EI_{zz} = 1$ and $L=1$



(c) Deflection curve for $w=0$,
 $P=0, M_C=100, EI_{zz} = 1$ and $L=1$