Assignment 0.2.1 : Reading and Practice problems

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1 Description

In order to remind you of some mathematical concepts which are relevant to this course (and to help you fill some holes in knowledge if you did not take certain mathematical courses), this homework will be to practice a few problems and to do some preparatory reading on greek letters, mathematical symbols and operators, and Matlab.

If you have trouble with the math problems, please come to office hours, email, IM, or review on your own the material. Start with going to mathworld, the link is on the course web page for quick definitions and references, then if still unsure, go to the library and check out some of the references there that are on our site.

There is an old saying when solving problems : *Garbage in equals garbage out.* We hope to avoid garbage in this class as much as possible by putting emphasis on the steps to solve things rather than just the final answer. Garbage is just plain smelly!

2 Reading

- *Read and review the brief pdf* ***Greek letters, Mathematical Symbols and Operators*** *handout downloadable from the course page handouts section at*

  http://maelabs.ucsd.edu/alex/pages/cogsci109

- *Read **Getting Started with Matlab**, pdf downloadable from the course handouts page, pages (1.2-1.8,2.1-2.32 , 4.2-4.7, 6.1-6.24).* It is quick reading, so it goes by fast. If you have access to Matlab from home, it helps to follow along with the reading, otherwise you could go to a computer lab with matlab installed (CSB115) - we will go over how to run matlab remotely (ie from a computer which does not have matlab installed)

3 Math problems - do at least 3 per section as completely as possible, but do a total of 12 problems minimum.

You may use matlab to perform the actual calculations (in fact you should gain practice by doing so), but you must write down all steps of the hand calculations, and list the matlab command you used with a one sentence description of what the command does (ie show your work, and know what the commands you use do).

If you complete all problems, show work and get more than 80**% of them correct, you receive an extra 5 points.**

For all Matlab plots, include axis labels and a title. Where appropriate include a legend as well.

3.1 Precalculus

3.1.1

$g(x) = x^3 + 2x^2 - 3$

Find: $g(0)$, $g(3)$, $g(-x)$, $g(1+h)$

$$g(0) = -3 \tag{1}$$

$$g(3) = 42 \quad (2)$$
$$g(-x) = -x^3 + 2x^2 - 3 \quad (3)$$
$$g(1+h) = h^3 + 5h^2 + 7h \quad (4)$$

3.1.2

Find the domain and range for:

a) $f(x) = 2x + 7,\ -1 < x < 6$

Domain: $x \in (-1, 6)$

Range: $y \in (5, 19)$

b) $g(x) = \frac{2}{3x-5}$

Domain $x \in (-\infty, \infty) \setminus \frac{5}{3}$

Range: $y \in (-\infty, \infty) \setminus 0$

3.1.3

Sketch a graph and find the domain

a) $f(x) = 3 - 2x$ Domain $x \in (-\infty, \infty)$

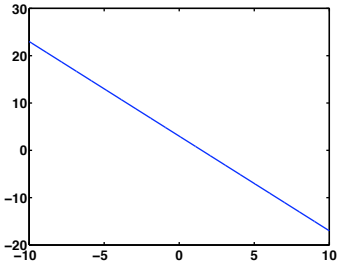

Figure 1: Plot for Problem 3.1.3.a

b) $f(x) = \frac{1}{x}$ Domain $x \in (-\infty, \infty) \setminus 0$

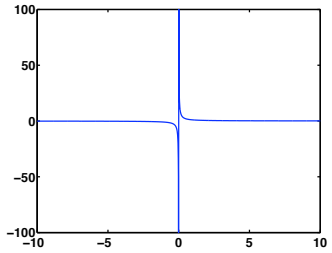

Figure 2: Plot for Problem 3.1.3.b

3.1.4

Use the definition of continuity and properties of limits to show that the function is continuous on the given interval. Use matlab to plot the function over the interval $[1, 100]$ in increments of 0.1. Use Matlab to include a title, and label the x and y axes, respectively.

$f(x) = x + \sqrt{x-1}$, $[1, \infty)$

Proving continuity: The definition of "continuous" is: "f is continuous at a only if $\lim_{x\to a} f(x) = f(a)$." By limit rules, we have:

$$\begin{aligned}
\lim_{x\to a} f(x) &= \lim_{x\to a} \left(x - \sqrt{x-1}\right) && (5)\\
&= \lim_{x\to a} x - \lim_{x\to a} \sqrt{x-1} && (6)\\
&= a - \sqrt{\lim_{x\to a}(x-1)}, (x-1) \geq 0 && (7)\\
&= a - \sqrt{\lim_{x\to a}(x) - 1}, x \geq 1 && (8)\\
&= a - \sqrt{(a-1)}, a \geq 1 && (9)\\
&= f(a), a \geq 1 && (10)
\end{aligned}$$

We also can show, using nearly identical logic that $f(a)$ is continuous from the right at the boundary point $a = 1$, and we can show that f has a well defined value at

$a = 1$, meaning the interval is closed on the right. $f(\infty) = \infty$, and so the interval is open on the left. But since every $a \in [1, \infty)$ follows the constraint $a \geq 1$, we have shown that $\lim_{x \to a} f(x) = f(a), x \in [0, \infty)$, and so f(x) is continuous on this interval.

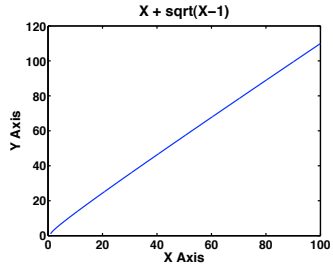

Figure 3: Plot for Problem 3.1.4

3.1.5

Explain why the function is discontinuous at the given point. Plot the graph of the function over an interval that contains the discontinuity using matlab. Use as fine a step size for the x variable as necessary to show the details of the shape of $f(x)$.

$f(x) = \frac{x^2-1}{x+1}$, $a = -1$

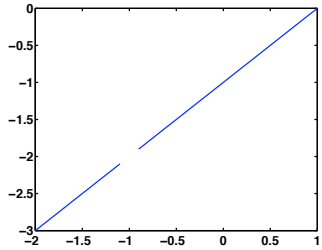

Figure 4: Plot for Problem 3.1.5

When $x = -1$, the denominator of $f(x)$ is 0. Functions with 0 in the denominator have an undefined value. The quotient $C = A/B$ is defined as "That which would multiply B to yield A." When $B = 0$ there is no value C that could be multiplied by

B to yield A, unless A is 0, in which case, any number C would satisfy this equation. For these reasons, we say that result of division is undefined (in general) when the denominator B is 0.

3.2 Calculus - Derivatives and Integrals

After finding the derivative (or integral as requested), plot using Matlab the function and derivative on the same page (as separate plots) using the *subplot* command, then plot over a reasonable range.

3.2.1

Find $f'(a)$ for $f(a) = x^3 + 3x$

$$f'(a) = 3x^2 + 3 \tag{11}$$

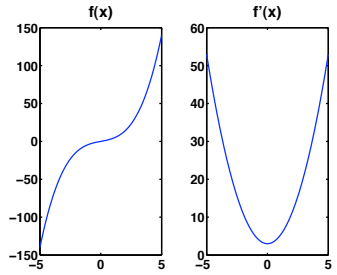

Figure 5: Plot for Problem 3.2.1

3.2.2

Find $f'(a)$ for $f(a) = \sqrt{x-1}$

$$f'(a) = \frac{1}{\sqrt{x-1}} \tag{12}$$

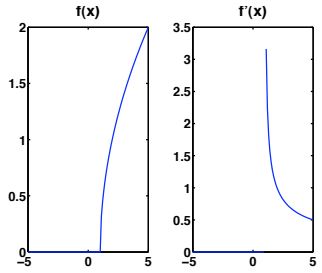

Figure 6: Plot for Problem 3.2.2

3.2.3

Find $f'(a)$ for $f(a) = \frac{x}{x^2-1}$

$$f'(a) = \frac{1}{x^2-1} - \frac{2x^2}{(x^2-1)^2} \tag{13}$$

$$= \frac{x^2+1}{(x^2-1)^2} \tag{14}$$

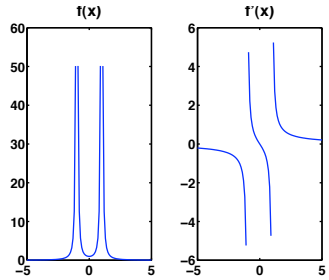

Figure 7: Plot for Problem 3.2.3

3.2.4

Find $f'(a)$ for $f(a) = x(x^3 - 3x + 11)$

$$f'(a) \quad = \quad 4x^3 - 6x + 11 \tag{15}$$

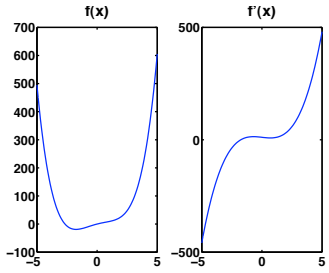

Figure 8: Plot for Problem 3.2.4

3.2.5

Find $f'(a)$ for $f(a) = 0.75 * \sin(x) + 0.25 * \cos(x) - x$

$$f'(a) \quad = \quad .75cos(x) - 0.25sin(x) - 1 \tag{16}$$

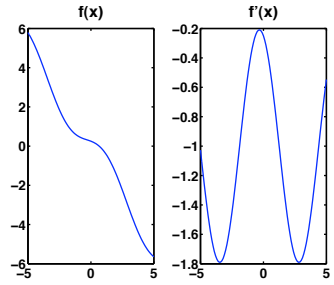

Figure 9: Plot for Problem 3.2.5

3.2.6

Find $f'(a)$ for $f(a) = \tan^2(x) + 0.1 * \sin(x^2)$

$$f'(a) = \quad = 2\tan(x)(1 + \tan^2(x)) + 0.2x\cos(x^2) \tag{17}$$

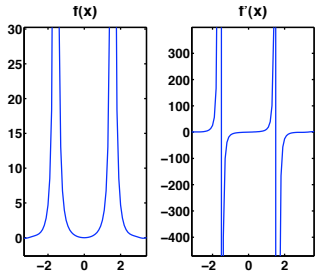

Figure 10: Plot for Problem 3.2.6

3.2.7

Find $f'(a)$ for $f(a) = 10^x + 5$

$$f'(a) \quad = \quad 10^x \ln(10) \tag{18}$$

3.2.8

Find $f'(a)$ for $f(a) = 6 * e^{-(3x-2)}$

$$f'(a) \quad = \quad -18e^{-(3x-2)} \tag{19}$$

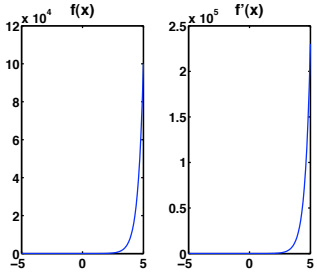

Figure 11: Plot for Problem 3.2.7

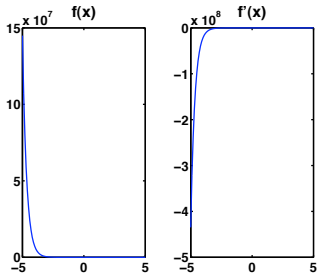

Figure 12: Plot for Problem 3.2.8

3.2.9

Find the integral of : $f(x) = x$

$$\frac{1}{2}x^2 + C \tag{20}$$

3.2.10

Find the integral of : $f(x) = \sin(10x)$

$$-\frac{1}{10}\cos(10x) + C \tag{21}$$

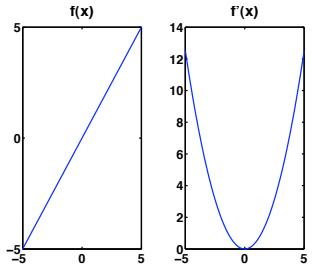

Figure 13: Plot for Problem 3.2.9

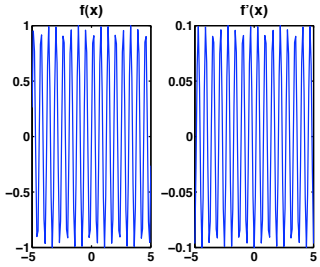

Figure 14: Plot for Problem 3.2.10

3.2.11

Find the integral of : $f(x) = e^x + e^{-x}$

$$e^x - e^{-x} + C \tag{22}$$

3.3 Matrices and vectors

Do this by hand, but use Matlab to check your work. Turn in a printout of the matlab command window showing the calculations (ie A=[1 0; 0 1]; B=blah; A*B...)

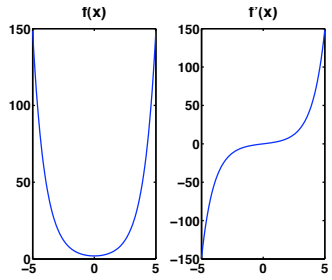

Figure 15: Plot for Problem 3.2.11

3.3.1

Perform the following mathematical operation

$x = \{1, 2, 3, 4, 5\}$

$y = \{-1, -5, -8, -3, 8\}$

$x + y =?$

$x + y = \{0, -3, -5, 1, 13\}$

3.3.2

Perform the following mathematical operation

$x = \{1, 2, 3, 4, 5\}$

$y = \{-1, -5, -8, -3, 8\}$

$x + y =?$

$x - y = \{2, 7, 11, 7, -3\}$

3.3.3

Perform the following mathematical operation

$x = \{1, 2, 3, 4, 5\}$

$\alpha = 100$

$\alpha * x =?$

$\alpha x = \{100, 200, 300, 400, 500\}$

3.3.4

Perform the following mathematical operation

$x = \{1, 2, 3, 4, 5\}$

$y = \{-1, -5, -8, -3, 8\}$

$x \bullet y =?$ (ie dot product)

$x \bullet y = -7$

3.3.5

Perform the following mathematical operation

$x = \{1, 2, 3, 4, 5\}$

$||x|| =?$ (ie norm)

$||x|| = \sqrt{55} = 7.4162$

3.3.6

Perform the following mathematical operation

$A = \begin{bmatrix} 2 & 5 \\ 8 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & -50 \\ 10 & 2 \end{bmatrix}$

$A + B =?$

$A + B = \begin{bmatrix} 3 & -45 \\ 18 & 4 \end{bmatrix}$

3.3.7

Perform the following mathematical operation

$A = \begin{bmatrix} 2 & 5 \\ 8 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & -50 \\ 10 & 2 \end{bmatrix}$

$A - B = ?$

$A - B = \begin{bmatrix} 1 & 55 \\ -2 & 0 \end{bmatrix}$

3.3.8

Perform the following mathematical operation

$A = \begin{bmatrix} 2 & 5 \\ 8 & 2 \end{bmatrix}$

$\alpha = 10$

$1/\alpha * A = ?$

$\alpha^{-1} * A = \begin{bmatrix} .2 & .5 \\ .8 & .2 \end{bmatrix}$

3.3.9

Perform the following mathematical operation

$A = \begin{bmatrix} 2 & 5 \\ 8 & 2 \end{bmatrix}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$A * x = ?$

$A * x = \begin{bmatrix} 2x_1 + 5x_2 \\ 8x_1 + 2x_2 \end{bmatrix}$

3.3.10

Perform the following mathematical operation

$A = \begin{bmatrix} 2 & 5 \\ 8 & 2 \end{bmatrix}$

$b = \begin{bmatrix} 3 \\ 10 \end{bmatrix}$

$A * b =?$

$A * x = \begin{bmatrix} 56 \\ 44 \end{bmatrix}$

4 List of useful Matlab commands and operators

Type *help functionname* to get help for the particular function (ie 'help plot')

- plot
- xlabel
- ylabel
- title
- axes
- clc
- close all
- clear
- % (ie commenting code)
- ;