CogSci 109: Lecture 11

Tuesday Oct 26, 2006

Non-Linear Interpolation, Spherical linear interpolation, Splines, Lagrange, error analysis

Announcements

- Handout on interpolation now has linear AND nonlinear methods
  - Look for more updates to it after today

Today we'll develop methods of nonlinear interpolation and extrapolation

- Lagrange
  - Useful for low number of data points
  - Unstable for high numbers of data points
- Splines
  - There are many kinds discussed in the reading, we'll just discuss one today
  - Good overall method
  - Works with many or few data points

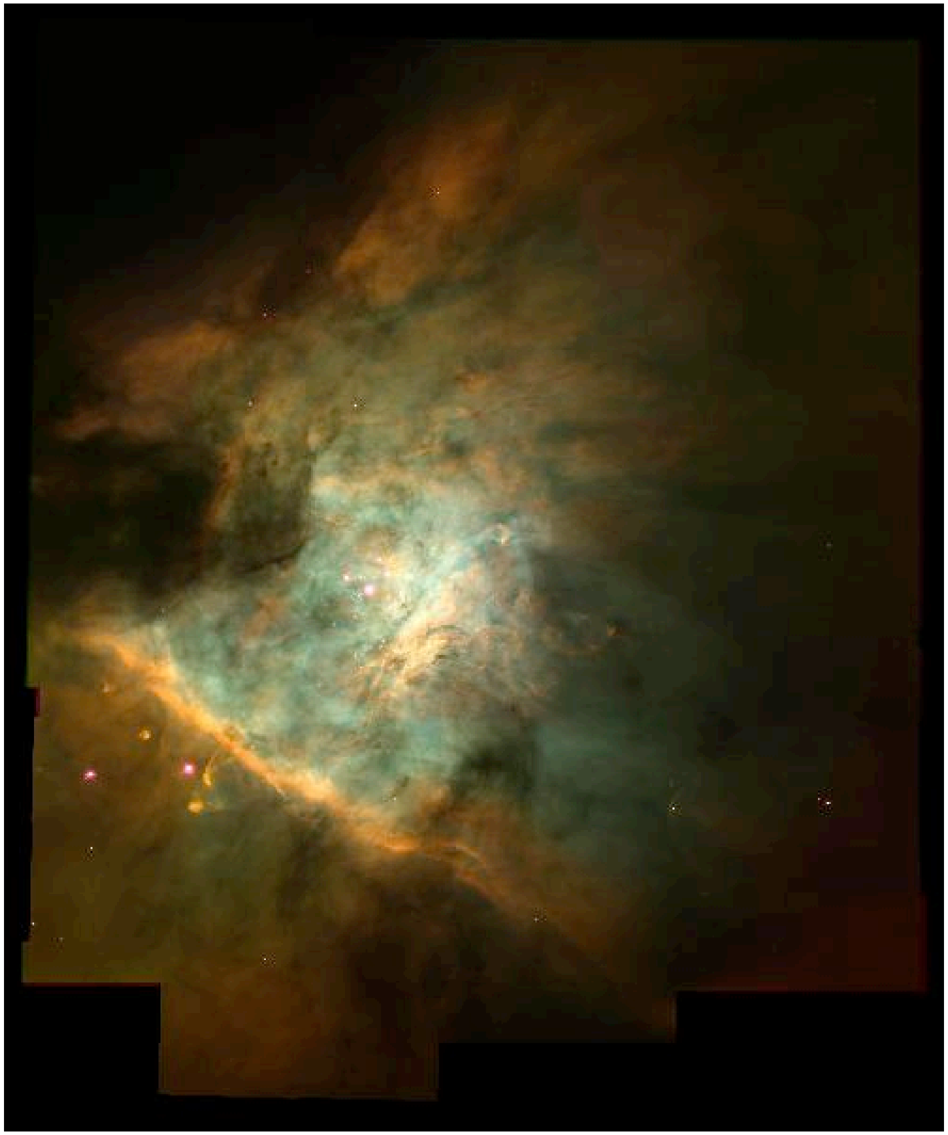
The importance of clear communication of data

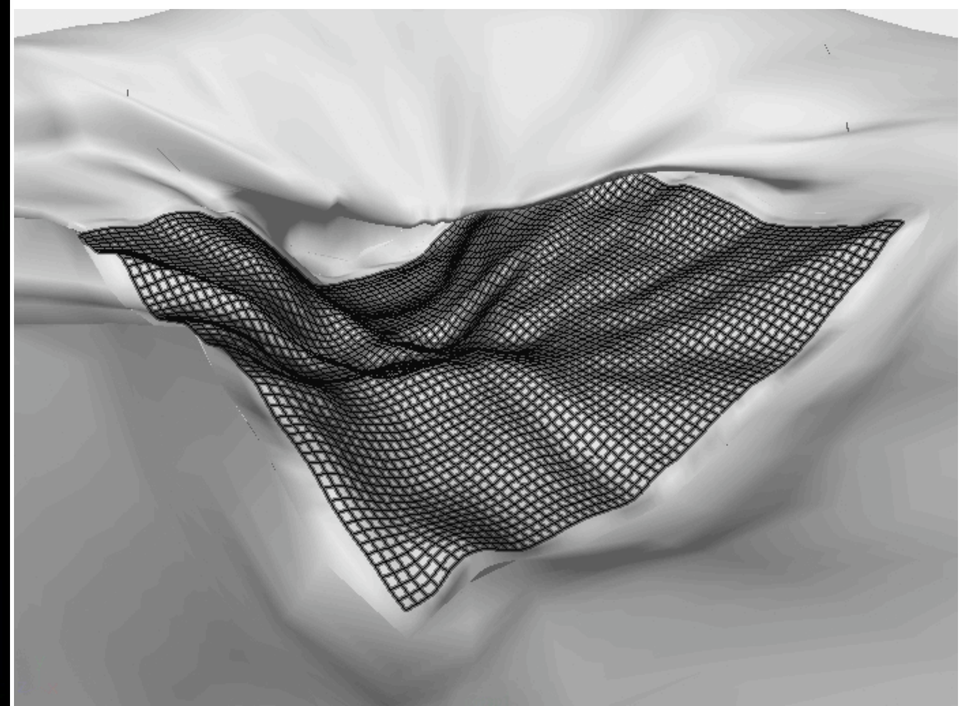
- See reading
- Different fields often have trouble communicating because of different basic concepts, mathematically or conceptually
- Highly mathematical fields have trouble communicating data to other fields because of these differences
  - Even within those fields
    - Example of control theory
- Being effective at communication will help you contribute and bring people together, whatever you do in life

S.L.E.R.P.

- Board completion of linear interpolation - spherical linear interpolation (SLERP)
  - Looking at things from different perspectives sometimes makes life much easier
  - Having trouble with something? Ask yourself the question 'Can I look at this from a different point of view?' And then 'How would I do that?'

Another SDSC example: the orion nebula animation





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Lagrange interpolation/extrapolation

- Fit a polynomial of degree that is the same as the number of points
  - If n points, degree of polynomial is n
- Makes a curve that exactly passes through all data points
- Use only for small number of data points

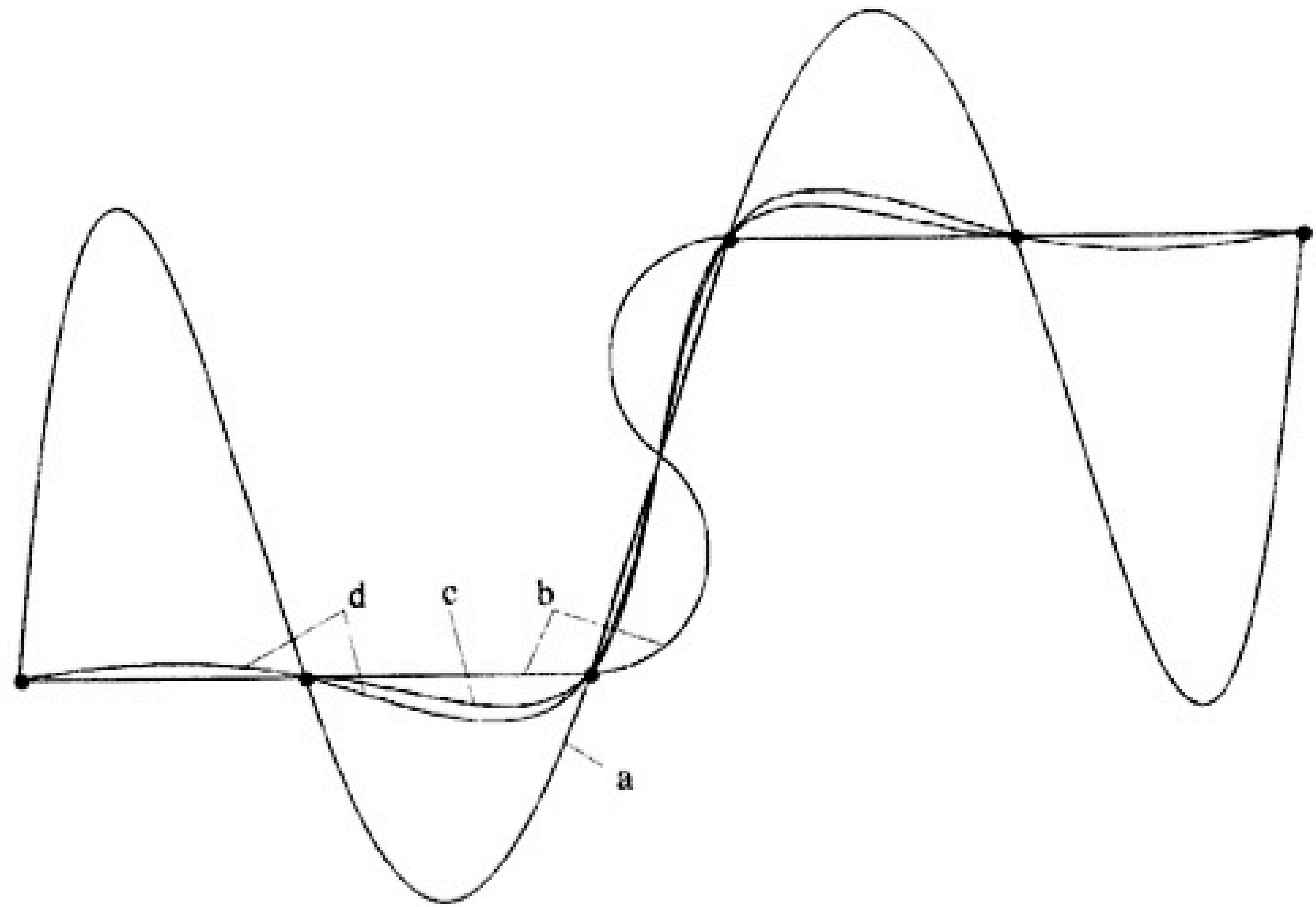
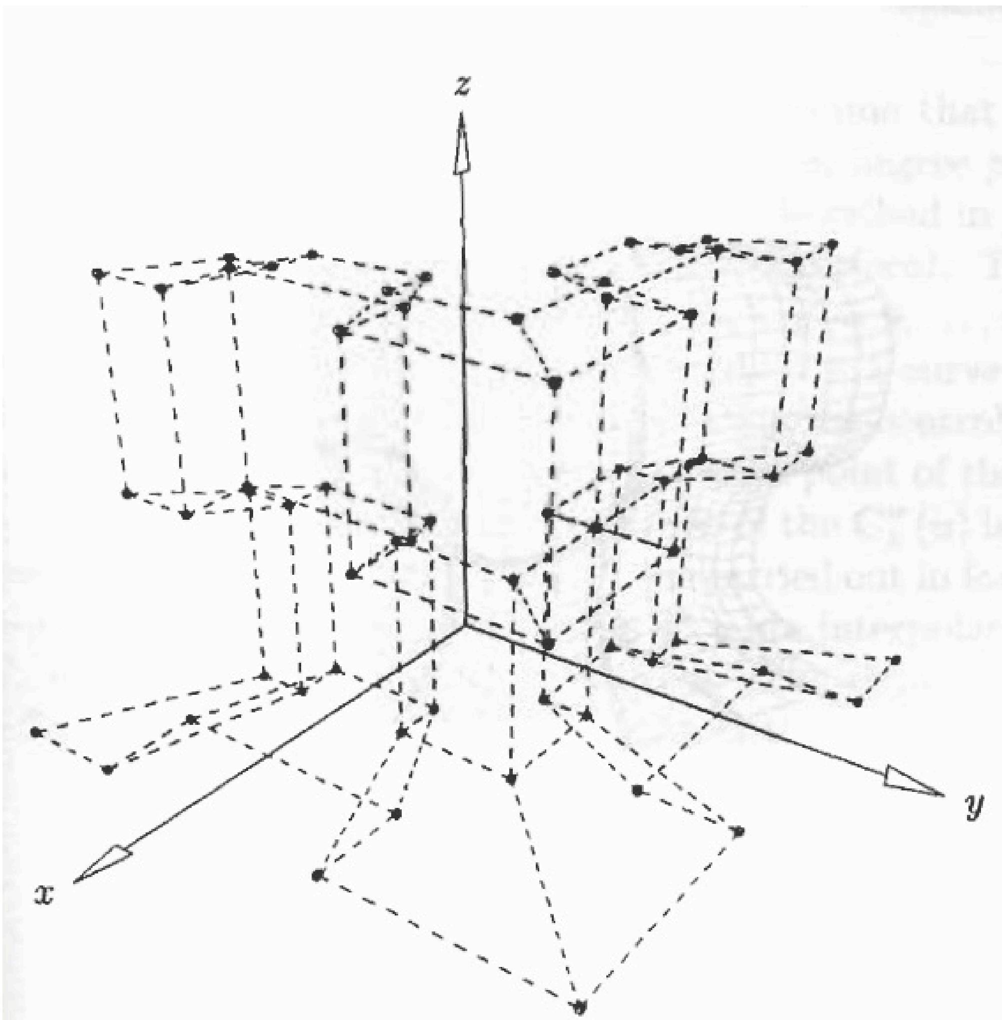


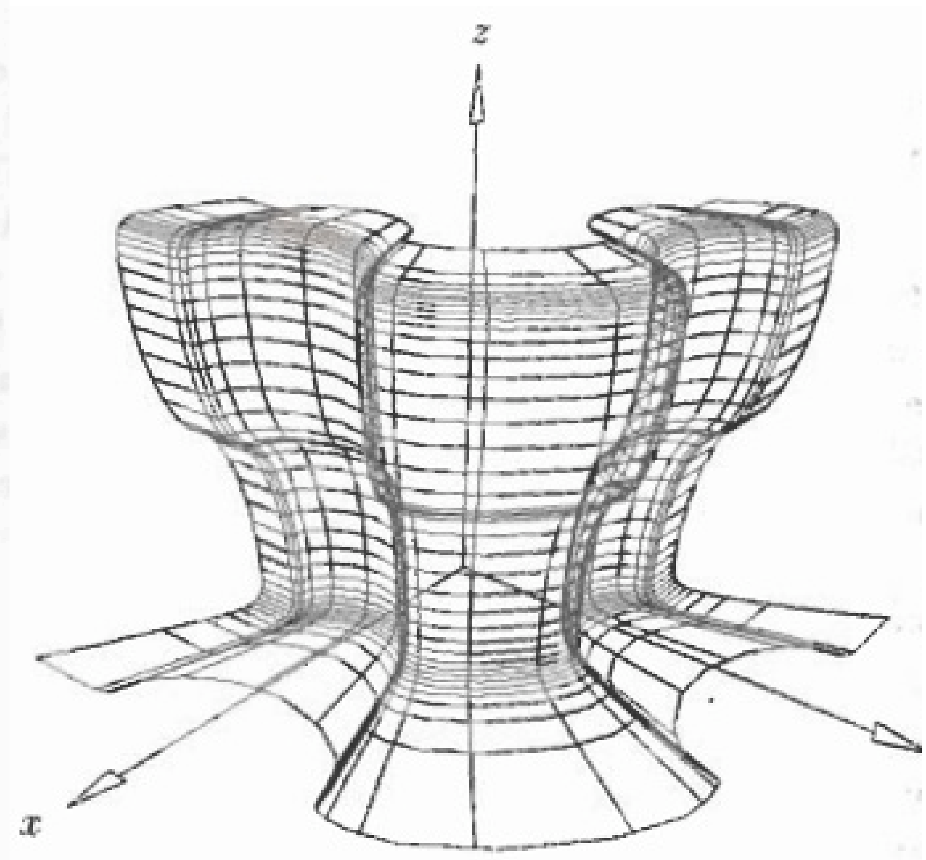
Figure 7.1 Interpolation curves drawn for six vertex points (dots), with y plotted in the vertical and x in the horizontal direction. Curves are shown for (a) a high-order polynomial fit, (b) a circular-arc fit, (c) a parabolic blend, and (d) a natural cubic spline.

Splines are useful in many places Lagrange fails

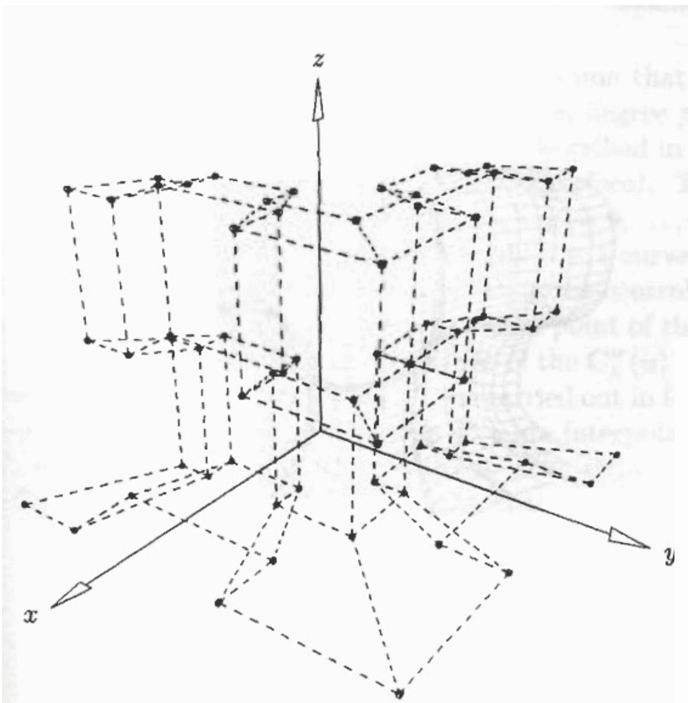
- Large number of data points
- Also can make a curve that passes through all data points
  - some types do not enforce this
- Drawn from drafting who drew from classical fine woodworking
  - Thin piece of wood stretched between pegs to create curves
  - Many types of splines dependent on end conditions
    - Pull tightly on the spline, curve gets sharper about the data points

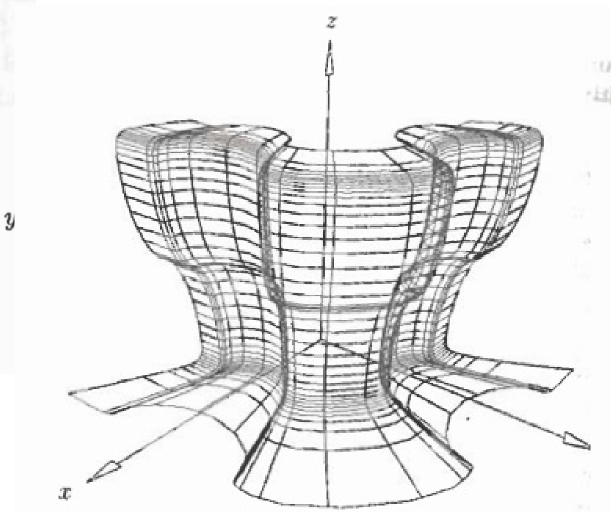
Splines are useful for N-Dimensions

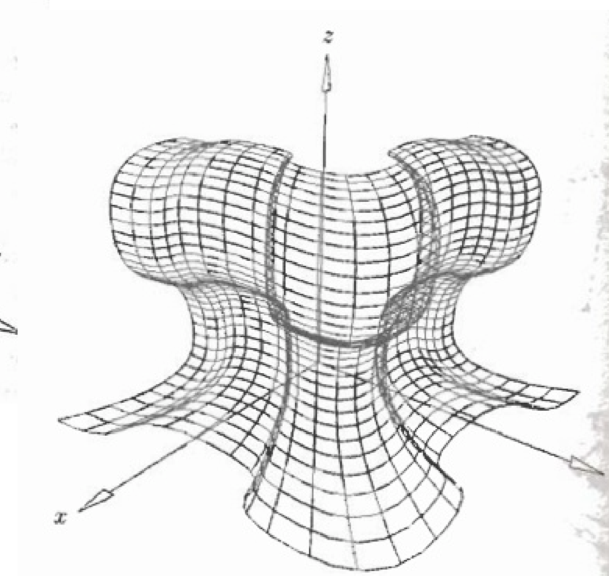




Splines also give you control over the final outcome of the curve





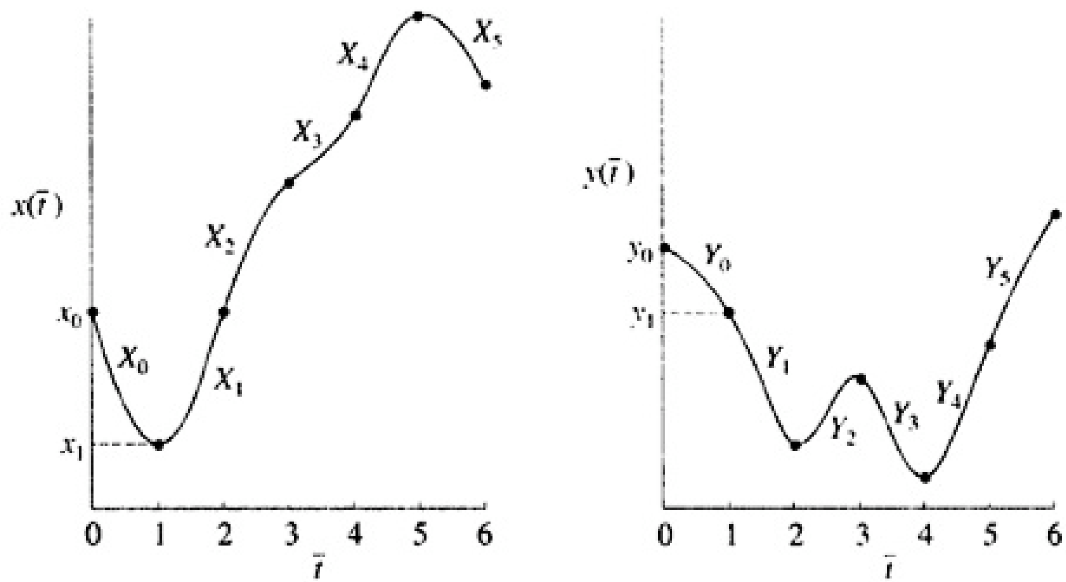


Some types of splines

- Natural cubic spline
- Quadratic B-Splines
- Hermite Cubic Splines
- Coons Cubic Splines
- Rational B-Splines
- NURBS (Non-Uniform Rational B-Splines)

Natural Cubic Spline - a conceptual introduction

- We construct the following curve in sections



Natural Cubic Splines

- We fit another parametric curve, with a value of t from 0-1 again and make the ith segment according to

$$Y_i(t) = a_i + b_i t + c_i t^2 + d_i t^3$$

- And we solve for each set of these constants by requiring continuity at the end points (one section smoothly flows into the next, and the slope must match as well)

$$Y_i(0) = y_i = a_i$$

$$Y_i(1) = y_{i+1} = a_i + b_i + c_i + d_i$$

$$Y_i'(0) = D_i = b_i$$

$$Y_i'(1) = D_{i+1} = b_i + 2c_i + 3d_i$$